Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Real Analysis

Subject Code: 4SC06RAC1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 23/04/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1	**Attempt the following questions:**	**(14)**
a)	Define : Lower bound of Sequence	**(01)**
b)	Write the range of the sequence $\{1+(-1)^n\}$.	**(01)**
c)	Define: Convergent sequence	**(01)**
d)	True or false: Every convergent sequence is bounded.	**(01)**
e)	Define: Infinite series	**(01)**
f)	Write the necessary condition for the convergent of series.	**(01)**
g)	Find $\overline{A_3}$ for the sequence $a_n = \left\{-2,-1,1,\frac{1}{2},\frac{1}{4},.....\right\}$.	**(01)**
h)	Define: p-series.	**(01)**
i)	State Darboux's theorem for integrals.	**(01)**
j)	True or false: f is bounded and integrable on $[a,b]$, if $\lvert f \rvert$ is bounded and integrable on $[a,b]$.	**(01)**
k)	Define: Primitive of the function	**(01)**
l)	What is norm of partition $P=\{1,1.1,1.4,1.5,1.6,1.75,1.85,2\}$ of $[1,2]$?	**(01)**
m)	True or false: If $U(P,f)\le S(P,f)$ then the function is Riemann integrable function.	**(01)**
n)	Define: $L(P,f)$	**(01)**

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	
a)	State and prove Bolzeno weiestrass theorem for sequence.	**(05)**
b)	State and prove first fundamental theorem of calculus.	**(05)**

c) Verify: $\inf a_n \le \underline{\lim}\, a_n \le \overline{\lim}\, a_n \le \sup a_n$ for the sequence $\left\{\dfrac{(-1)^n}{n^2}\right\}$. **(04)**

Q-3 Attempt all questions

a) Define: Oscillation of $f(x)$ in $[a,b]$ **(02)**

b) Prove that any bounded and monotonic sequence is convergent **(04)**

c) Which of the following sequences are convergent, divergent, oscillating finitely, oscillating infinitely, bounded and unbounded? **(08)**

1) $\left\{-2,-1,0,\dfrac{1}{2},\dfrac{1}{3},\dfrac{1}{4},....\right\}$ 2) $\{n^2\}$

3) $\left\{\dfrac{(-1)^{n-1}}{n!}\right\}$ 4) $\left\{\left(1+\dfrac{1}{n}\right)^n\right\}$

Q-4 Attempt all questions

a) Which of the two basic series we consider for the comparison test of kind one? **(02)**

b) Show that the sequence $\left\{\left(1+\dfrac{1}{n}\right)^n\right\}$ is convergent and $\lim\limits_{n}\left(1+\dfrac{1}{n}\right)^n$ lies between 2 and 3. **(05)**

c) State and prove Cauchy's integral test for convergence of series. **(07)**

Q-5 Attempt all questions **(14)**

a) State Cauchy's general principle of convergence of sequence. **(02)**

b) Show that $f(x)=3x+1$ is Riemann integrable on $[0,k]$ and $\int_1^2 f(x)\,dx=\dfrac{11}{2}$. **(04)**

c) Test the convergence of the following series. **(08)**

1) $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+....$ 2) $\sum\dfrac{(-1)^{n-1}}{n^2}$

3) $\sum\dfrac{x^n}{n}$ 4) $\dfrac{1^2\cdot 2^2}{1!}+\dfrac{2^2\cdot 3^2}{2!}+\dfrac{3^2\cdot 4^2}{3!}+.....$

Q-6 Attempt all questions **(14)**

a) State and prove Leibnitz test for alternating series. **(06)**

b) Write the statement of the following test for the convergence of series: **(04)**

1) D'alembert's ratio test
2) Raabe's test

c) Show that $\sum\dfrac{(-1)^{n-1}}{\log(n+1)}$ is conditional convergent. **(04)**

Q-7 Attempt all questions **(14)**

a) Define : Riemann lower sum of the function on [a,b] **(02)**

b) If f is integrable on $\left[a,b\right]$ then prove that f^{2} is integrable on $\left[a,b\right]$. **(05)**

c) If f_1 and f_2 are two integrable functions on $\left[a,b\right]$ then prove that f_1+f_2 is also integrable on $\left[a,b\right]$ and $\int_a^b (f_1+f_2)\,dx=\int_a^b f_1\,dx+\int_a^b f_2\,dx$. **(07)**

Q-8 Attempt all questions

a) If $\{a_n\}$ be any sequence then $\underline{\lim}(-a_n)=-\overline{\lim}(a_n)$ **(02)**

b) Define absolutely convergent series and show that the series $\sum\frac{(-1)^{n-1}}{n\cdot 2^{n}}$ is absolutely convergent. **(05)**

c) Show that $\int_0^t \sin x\,dx=1-\cos t$. **(07)**